A STUDY OF THE RELATIONSHIP BETWEEN CONCEPTUAL TEMPO AND PROBLEM-SOLVING ABILITIES OF FOURTH-GRADE CHILDREN

By

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To

My Parents, Mary, and Joan

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By

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August, 1972

Chairman: Dr. Kenneth P. Kidd
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Major Department: Curriculum and Instruction

Purpose of the Study

The purpose of this study was to determine the relationship between a fourth-grade child's impulsive-reflective response style and two aspects of his problem-solving ability --- namely, his ability to select the proper arithmetic operation in a verbal arithmetic problem and his ability to estimate.

Procedures

The sample population for the study consisted of the entire fourth-grade enrollment of three public elementary schools in Alachua County, Florida. On the basis of their scores on the arithmetic subtest of the Stanford Achievement Test, Intermediate I, Form X, a group of high achievers and a group of low achievers were identified. From these two available pools, 63 high achievers and 63 low

achievers were selected at random. The Matching Familiar Figures Test, a conceptual tempo assessment scale developed by Dr. Jerome Kagan, was administered individually to each subject in the two randomly selected groups. Hence, four main categories emerged from this selection process: impulsive high achievers, impulsive low achievers, reflective high achievers, and reflective low achievers.

A performance test on two aspects of mathematical problem-solving ability was developed by the author. Section A of the test consisted of a series of one-step verbal arithmetic problems designed to assess a student's ability to analyze such a problem and select the appropriate arithmetic operation needed to solve it. Section B contained a series of problems involving the ability to estimate in various situations. The test was administered by the author to all fourth-graders in the three schools, and the resulting error scores of the subjects in the four categories mentioned previously served as the criterion measure for the study. The resulting mean error scores of the four groups were compared in a 2x2 factorial design using the methods of multiple linear regression analysis.

Conclusions

In comparisons of impulsive and reflective subjects, both without regard to achievement status and also within the two achievement categories, the reflective subjects did significantly better than the impulsive subjects on

Section A and on the entire test. The results on Section B did not reach statistical significance, but in each comparison of impulsive and reflective subjects, the impulsive subjects appeared to have a slightly smaller mean error score on the estimation tasks.

The performance of high achievers, both without regard to response style and within the impulsive category, was significantly better than that of low achievers on Sections A and B, and the entire test. Reflective high achievers did significantly better than reflective low achievers on Section A and the entire test. While the results on Section B did not reach statistical significance, it again appeared that reflective high achievers did slightly better.

There were no significant differences between subject groups having different patterns of attributes with respect to conceptual tempo and achievement. The interaction of conceptual tempo and achievement status did not make a statistically significant difference in error scores on Sections A and B, and the entire test.

CHAPTER I

INTRODUCTION

General Background of the Problem

As far as mathematics educators are concerned, the last fifteen years can be characterized as a period of intense activity, much of it on a national scale, directed toward the improvement of the mathematics curriculum of the elementary and secondary school. Teams of mathematicians and mathematics teachers have worked together in various curriculum reform projects, to develop new course content and achieve a better, more effective organization of existing content.

In addition to these changes, there has emerged from all this activity, an increasing emphasis on student understanding and active student involvement in the learning process, usually through some type of discovery activity. This emphasis on student understanding has called for more thinking and hence more use of problem-solving activities on the part of the student. Hence the question of how the mathematics teacher can most effectively promote good mathematical problem-solving behavior among his students has assumed added significance.

Many of the research studies which have dealt with this question during the last ten years, have involved a comparison of two or more problem-solving strategies in an effort to determine which is more effective in solving a specific problem situation. The results from these studies have been limited in scope, fragmented, and often contradictory. In an effort to develop a more unified approach to the problem, several recent studies have attempted to examine certain traits and characteristics of the individual and then relate these to problem-solving ability. Additional studies of this type are needed, however, before the effectiveness of such an approach can be determined. It was in response to this need that the present study was undertaken.

Statement of the Problem

The purpose of this study is to determine the relationship between a fourth-grade child's impulsive-reflective response style and two aspects of his problem-solving ability --- namely, his ability to select the proper arithmetic operation in a verbal arithmetic problem and his ability to estimate. In particular, as a prelude to the formal null hypotheses found in Chapter III, the following research questions are of interest:

1. Do fourth-grade students who have a reflective response style perform significantly better on estimation tasks than fourth-grade students who have an impulsive response style?

2. Do fourth-grade students who have a reflective response style perform significantly better in identifying arithmetic operations in word problems than fourth-grade students who have an impulsive style?
3. Do fourth-grade students having a reflective response style, who have also been identified as high achievers, perform significantly better on estimation tasks than fourth-grade students who have an impulsive response style and have been identified as high achievers?
4. Do fourth-grade students having a reflective response style, who have also been identified as high achievers, perform significantly better in identifying arithmetic operations in word problems than fourth-grade students who have an impulsive response style and have been identified as high achievers?
5. Do fourth-grade students having a reflective response style, who have also been identified as low achievers, perform significantly better on estimation tasks than fourth-grade students who have an impulsive response style and have been identified as low achievers?
6. Do fourth-grade students having a reflective response style, who have also been identified as low achievers, perform significantly better in identifying arithmetic operations in word problems than fourth-grade students who have an impulsive response style and have been identified as low achievers?

In order to test the series of null hypotheses generated by these research questions, 93 fourth-grade students from the public schools of Alachua County, Florida, were identified as impulsive high achievers, impulsive low achievers, reflective high achievers, or reflective low achievers on the basis of the Stanford Arithmetic Achievement Test and the Matching Familiar Figures Test. A performance test on two aspects of problem-solving ability was administered to the subjects and the resulting mean error scores of the four groups were compared in a 2x2 factorial design using the methods of multiple linear regression.

Definition of Terms

The following terms will be used throughout the study:

Reflective Subject: a subject who is above the group median on latency of response, and below the group median on errors on the Matching Familiar Figures Test.

Impulsive Subject: a subject who is below the group median on latency of response, and above the group median on errors on the Matching Familiar Figures Test.

High Achiever: a subject whose stanine score on any two of the three parts of the Stanford Arithmetic Achievement Test was five or above, and whose stanine was three or above on the remaining part.

Low Achiever: a subject whose stanine score on any two of the three parts of the Stanford Arithmetic Achievement Test was three or below, and whose stanine score was four or below on the remaining part.

Need for the Study

One particular area of concern for teachers of mathematics has been that of helping the student to improve his problem-solving ability. Although most mathematics educators readily acknowledge this as one of the goals of mathematics instruction, there is a noticeable lack of agreement as to how this goal can best be accomplished. J. W. Getzels, a leading psychologist in the area of problem-solving and creative thinking, made the following statement: "It is evident that no single set of principles of instruction for creative thinking and problem-solving can be drawn from present theory and research. The theory is too diverse, the research too scant." (1964, p. 241) In an article for the Review of Educational Research, Kilpatrick (1969) reviewed the research on problem-solving for a recent five-year period and concluded that problem-solving is not presently being systematically investigated by mathematics educators. Most of the studies he examined were designed chiefly to investigate the relative effectiveness of specific methods of teaching problem-solving and few were built on any previous research or had an explicit theoretical rationale. According to Kilpatrick, more useful information about the nature of mathematical problem-solving ability might be obtained by examining the relationship between an individual's success in problem-solving and other characteristics of his thinking and personality. Several of the more recent research studies reviewed by Kilpatrick indicate that some mathematics

educators are beginning to borrow ideas and techniques from recent psychological studies of higher cognitive processes, and Kilpatrick feels that such an approach may give research on problem-solving in mathematics some direction and cohesiveness.

According to both Getzels and Kilpatrick then, there is an apparent need for systematic basic research in problem-solving, especially for studies which attempt to relate success in problem-solving to personality factors. The present study is of this latter type. The research studies reviewed in Chapter II indicate that the existence and significance of a dimension of reflection-impulsivity in problem situations has been experimentally verified and some information about the relation of this response dimension to age, stability over periods of time, and generality across tasks has been collected. Most of these investigations, however, have been conducted in nonmathematical settings, so that little or no information exists relating conceptual response tempo to mathematical problem-solving ability. The present study attempts to add to the general fund of research information on conceptual tempo by investigating its relationship to certain aspects of mathematical problem-solving. If conceptual tempo does, in fact, generalize to mathematical problem-solving tasks, then the mathematics teacher can make use of this additional information to more effectively adapt his instructional methods to the needs and characteristics of the individual student.

Many writers have outlined various general schemes or sequences of steps which they feel can be utilized as guidelines to effective problem-solving in mathematics. [Brown (1964), Georges (1956), Polya (1962), Johnson and Rising (1967), Dewey (1933), Henderson and Pingry (1953)] Although these problem-solving strategies are many and varied, certain common elements can be discerned. Two of these --- the ability to select the proper operation in a verbal arithmetic problem and the ability to estimate a solution --- were chosen as the two problem-solving skills to be investigated in the present study. Both are considered by most writers to be important aspects of any problem-solving strategy.

The basic difficulty encountered by the student in solving a verbal problem is the translation of the situation into a number question or an open mathematical sentence. In order to do this, he must analyze the given situation, sort out the usable facts, state the question and identify the needed mathematical symbols. He must then determine the relationship which exists between the facts and the unknown, and according to this relationship, arrange the symbols in the form of a mathematical sentence or equation. Whether or not this equation is correct, depends to a large degree, on the student's ability to perceive which operation or sequence of operations must be utilized to obtain the unknown quantity from the given facts. In many problem situations, the needed operation is obvious to the student upon a first reading of

the problem. The ability to select an appropriate operation plays a more significant role, however, in those problem situations which involve operations which are frequently confused. For example:

1. Bob is 14 years old, Jim is 7. Bob is how many times as old as Jim? (Division)
2. Bob is 14 years old, Jim is 7. Bob is how much older than Jim? (Subtraction)

Proficiency in estimating the answer to a mathematics problem prior to actually working the problem is also a problem-solving characteristic which is considered by many mathematics educators to be an important instructional objective. In a problem situation in which many response alternatives are available, the ability to effectively estimate the solution before working the problem helps the student reduce the number of alternative responses and concentrate his efforts on those solutions of the same general type as the estimated solution. Estimating the solution can also provide the student with a form of feedback on the validity of his reasoning and calculations. If the calculated answer differs markedly from the originally estimated solution, the student then realizes that a re-examination of his calculations is in order.

Organization of the Study

Chapter I has been an introduction to the study, including some general background information, a statement

of the problem, definitions, and an explanation of the need for the study. Chapter II is devoted to a review of related research. The results from a series of eight studies concerning conceptual tempo are examined, along with the results from eight studies dealing with specific aspects of problem-solving ability. Chapter III contains the formal null hypotheses, along with a description of the design, the sample population, and the procedures involved in gathering the data. Information about each of the three instruments used in the study is presented and the statistical treatment is explained. Chapter IV is devoted to a presentation and analysis of the data. Chapter V contains a brief summary of the study together with a list of the conclusions reached. Several limitations are cited, and some implications for instruction and future research are discussed.

CHAPTER II

REVIEW OF RELATED RESEARCH

It has long been an assumption of many researchers involved in the study of cognitive development that the observable differences in the cognitive products of children of the same age and also of children of different ages could be attributed to factors such as availability of vocabulary and knowledge of deductive or inductive rules. Since this assumption is both intuitively sound and empirically verifiable, there has been little serious interest in identifying other factors which may contribute to age and individual differences in determining the form and quality of cognitive products. A series of recent investigations by Kagan and others [Kagan (1965a), (1965b), (1966a), (1966b); Kagan, Moss, and Sigel (1963); Kagan, Pearson, and Welch (1966); Kagan, Rosman, Bernice, et. al. (1964)] have discerned a fairly stable cognitive dimension referred to as "conceptual tempo." Essentially, this dimension describes the degree to which a child reflects upon alternative classifications of a stimulus or alternative solution hypotheses in situations in which many response possibilities are available simultaneously. In such situations, some children have a fast conceptual

tempo. They impulsively report the first classification that occurs to them or act upon the first solution sequence that seems appropriate. On the other hand, reflective children characteristically delay before reporting a classification or carrying out a solution scheme. These children tend to consider available alternatives and compare their validity.

The present interest in the impulsive-reflective response disposition of children developed as an outgrowth of earlier interest in analytic classification and conceptualization habits among normal subjects. As a result of some work with adult subjects at the Fels Research Institute, Dr. Jerome Kagan discovered that some of the subjects exhibited a preference for analytic conceptual groupings, that is, groupings based upon shared similarity in a particular objective component among a set of stimuli.

Kagan, Moss, and Sigel (1963) reported that a series of similar investigations with school-age children revealed that some children also exhibited a clear preference for grouping familiar objects together on the basis of objective elements of similarity that were component parts of the total stimulus. The technique used to measure this preference for analytic conceptualizations was a set of thirty cards, each with three black and white drawings of familiar objects, called the Conceptual Style Test (CST). The child was instructed to group two of the three pictures together and state the basis for his grouping. The results of the study

indicated that children who were highly analytic on the CST, in comparison to nonanalytic children, were more likely to (a) produce interpretations of ink blots that were highly differentiated and contained references to descriptive aspects of the stimulus, (b) attach a new verbal response to the component parts of a geometric design rather than only to the most salient aspect of the stimulus, and (c) inhibit impulsive and uncontrolled motor behavior. In addition, the study indicated that the tendency to delay a conceptual decision was associated with the production of analytic concepts on the CST.

In another study, Lee, Kagan, and Rabson (1963) demonstrated that analytic boys acquired concepts requiring analysis of picture stimuli earlier than nonanalytic boys.

Kagan, Rosman, Day, Albert, and Phillips (1964) report the results of a series of eight studies involving children in grades one to four, which were designed to inquire into the immediate and historical determinants of a preference for analytic conceptual groupings. These results revealed that two more fundamental cognitive dispositions each contributed variance to the production of analytic concepts, namely the tendency to reflect over alternative solutions or classifications in situations in which several response alternatives are available simultaneously, and the tendency to analyze visual arrays into their component parts. These two dispositions were relatively independent of each other, orthogonal to verbal skills, and each influenced the

frequency of errors in perceptual recognition tasks. Degree of reflection over alternative solution hypotheses (as measured by response time) displayed remarkable generality across a variety of tasks and marked intra-individual stability over a one-year period.

One of the experimental studies in this article also provided evidence which suggested that the tendency to inhibit impulsive, inaccurate answers to problems with alternative solution possibilities was also amenable to experimental manipulation. The subjects were 180 second-grade children who were assigned to one of two experimental groups and administered three conceptual tests: Conceptual Style Test (CST), Design Recall Test (DRT), and Hidden Figures Test (HFT). One group was instructed to respond as quickly as possible, while the other group was told to respond slowly and to reflect upon their answers. Children in the slow-instruction group produced more analytic concepts on CST and made fewer errors on DRT, suggesting the validity of the hypotheses that a set to reflect would increase analytic concepts and decrease perceptual recognition errors, while a set to respond quickly would have the opposite effect.

Still another of this group of studies assessed the effect of impersonal versus reassuring testing conditions on the reflection-impulsivity dimension among third-grade children. There was no consistent difference in errors or response time on three different tasks between subjects examined under "cold versus warm" testing conditions. However,

there was marked intra-individual consistency in response time and errors across the three tasks, supporting the generality of this cognitive disposition.

A year later, Kagan (1965b) reported the results of a study intended to assess the relationship between the reflection-impulsivity dimension and reading ability in primary grade children. Sixty-five boys and 65 girls in the first grade in three public schools were administered measures of reading skill and indexes of reflection-impulsivity (in the form of design-matching tests) in grade one and again at the end of grade two. Children who were impulsive, in contrast to reflective, in the selection of response alternatives in these design-matching tests (i. e. made many errors and displayed fast decision times) made more recognition errors in reading English words presented singly or in a prose selection. The relation between fast decision times and reading errors was better for high-verbal than for low-verbal children. The measures of impulsivity on design-matching tests gathered in grade one were positively correlated with reading errors one year later, with fast response times being the better predictor for boys and high error scores being the better predictor for girls.

Still another study by Kagan, Pearson, and Welch (1966) attempted to relate conceptual impulsivity and inductive reasoning. Seventy-nine boys and 76 girls in the first grade in Newton, Massachusetts, were administered test batteries consisting of measures of reflection-impulsivity, assessment of

performance on three inductive reasoning tests, and the information and vocabulary scales from the Wechsler Intelligence Scale for Children. The hypothesis being tested was that impulsive children would make more errors than reflective children of similar verbal ability on inductive reasoning problems, for they would be prone to select an answer which was not carefully evaluated. The results supported this conjecture for it was found that impulsive children had faster response times and higher error scores on the inductive reasoning tests than reflective children, even when verbal ability was controlled.

Callahan and Passi (1971) incorporated conceptual tempo into a study designed to assess the relationship between the impulsive-reflective dimension and ability to conserve length on certain Piagetian tasks. Thirty-three kindergarten and 39 first-grade children were administered two conservation-of-length tasks and an impulsive-reflective scale. A chi-square analysis of the data indicated no statistically significant relationship between the two characteristics. However, the more reflective children did seem to do better on conservation on the Piagetian tasks than the impulsive children. Post hoc point-biserial correlations were then carried out, relating both errors on the scale and response latency to ability to conserve length. Again the results favored the reflective children but failed to reach statistical significance.

Several studies of factors which affect mathematical

problem-solving ability have been done, and deserve to be mentioned.

Sherrill (1971) investigated the effect of different methods of presenting mathematical word problems. A testing instrument was constructed using selected problems from the National Longitudinal Study of Mathematical Abilities (NLSMA) and tenth-grade mathematics textbooks. Form A consisted of twenty mathematical word problems each having five distractors, and was administered to 114 tenth-grade students. Form B was similar in form to Form A, except that each problem was accompanied by an accurate pictorial representation of the problem situation. It was given to 96 tenth-grade students. Form C was similar to Form B except that the pictorial representations were distorted. It was given to 112 fourth-graders. Sherrill concluded that a subject's achievement with mathematical word problems is affected by the method used to present the problem. Other factors such as I. Q., reading ability, and grade average in the preceeding year's mathematics courses also were found to have some effect on achievement.

Another study which focused on characteristics related to problem-solving ability was conducted by Dodson (1971). The purpose of the study was to describe successful insightful mathematical problem-solvers in terms of (1) their performance on mathematics achievement tests, (2) their possession of certain cognitive and personality traits, (3) the characteristics of their teachers, and (4) the characteristics

of their schools and communities. Data on 77 variables were obtained from the data bank of the National Longitudinal Study of Mathematical Abilities. Statistical analyses were performed to determine which variables discriminated among six ability groups formed on the basis of performance on a criterion test. From this study, Dodson formed a composite list of the strongest characteristics, aside from superiority in mathematics, of successful problem-solvers. Some of these were:

1. high scores on verbal and general reasoning tests,
2. success in determining spatial relations,
3. ability to resist distractions, to identify critical elements, and to disregard irrelevant elements,
4. being a divergent thinker, and
5. positive attitude toward mathematics.

Boyden (1970) constructed a computer-scored diagnostic test in verbal problem-solving. She administered survey tests to 993 fifth-grade students in eight schools in Dade County, Florida, and tabulated the errors into categories. The tests indicated that problems involving measurement presented the most difficulty, followed by a tendency to focus upon numbers rather than ideas, and failure to complete the second step in a two-step problem. The results of the survey and diagnostic tests justified the conclusion that errors in verbal arithmetic problem-solving can be isolated, diagnosed, and analyzed through computer treatment of data. Boyden also concluded that further research was needed to produce more refined

instruments and computer programs, and to help identify pupil characteristics responsible for errors in verbal problem-solving.

Another study by Hyer (1971) examined the relationship of three cognitive styles to problem-solving. These styles were (1) response tempo, as measured by the Sutton-Smith, Rosenberg Impulsivity Scale for Children, (2) response style, as measured by the Perceptual Reaction Test, and (3) response ambiguity, as measured by the Modified Revised California Inventory. The Otis-Lennon Mental Ability Test was also used as a measure of intelligence for the subjects in the study. Three verbal reasoning ability tests were administered to 288 seventh- and eighth-graders, and the scores on these tests constituted the problem-solving criterion variables in the study. Six research hypotheses were formulated, proposing that the three cognitive styles, both exclusive of and in combination with intelligence, were significantly related to the three forms of problem-solving. Hyer concluded that response ambiguity and response style were significantly related to a composite problem-solving score. When intelligence was considered, however, it overshadowed and effectively masked the relationship of cognitive styles to problem-solving.

Swart (1970) compared the performance on various problem-solving tasks of two groups of fourth-graders. The groups differed in the amount of instruction they received on problem-solving. The low structure group received a

minimal amount of instruction in problem-solving techniques. For a given problem, a child from this group was simply required to draw a picture of the situation which told the whole story. No arithmetic was to be done. The adequacy of the picture was taken as evidence that the child had solved the problem. The high structure group received instruction in writing equations in order to solve verbal problems. The results of the study indicated that for usual fourth-grade textbook problems, children who had not been introduced to a systematic scheme for solving verbal problems did as well as those who learned to write equations. In area measurement, the low group did significantly better than the high group. In transferring area measurement concepts from rectangles and triangles to parallelograms, the low group showed some superiority, but this was not significant. Finally, subjects in the low group exhibited a greater affinity for verbal problems than those in the high group, when placed in a free choice situation on an affinity test.

Kramer (1966) states that teachers often tend to overemphasize computation at the expense of problem-solving in mathematics. He points out that more than just computational skill is needed for the successful solution of word problems. A student may master mathematical skills without being able to apply them because of his inability to understand the meaning of the fundamental arithmetic operations and the interrelationships among those operations.

Pace (1961) conducted a study in an attempt to

determine the effect understanding the fundamental operations has upon problem-solving ability. A control group and an experimental group were formed using students from the fourth grade of a training school. The groups were equated with respect to chronological age, mental ability, and performance in arithmetic reasoning and computation. During an eight-week experimental period, both groups of students worked through a series of twenty-four problem sets. Each set contained two one-step verbal problems for each of the operations of addition, subtraction, multiplication, and division. The children in the experimental group had to read each problem, tell how it was to be solved, and defend their choice of the operation. Those in the control group merely solved the problems without any discussion of their work. At the end of the experimental period, an alternate form of the arithmetic reasoning test used as a pretest was administered. Four special problem tests were also administered at spaced intervals during the experimental period. Each test contained ten problems of the conventional variety and five problems with distorted cues. Results of the study show that gains in performance on the arithmetic reasoning test were negligible for the control group but were statistically significant for the experimental group. Both groups showed improvement from Test 1 to Test 4, but the experimental group showed greater improvement than the control group. From his study, Pace concludes that the development of understanding of the four fundamental operations is a vital factor as far as improvement in problem-solving is concerned.

In another study, Kliebhan (1955) looked at the arithmetic problem-solving ability of sixth-grade boys. His results indicated that the principal difference between high and low achievers originated in differences in ability to estimate answers to problems.

Summary

In general, the results of the studies which have been reviewed in this section support the existence and significance of a dimension of reflection-impulsivity in problem situations where many solution hypotheses are available simultaneously, and the child has to evaluate the differential adequacy of each possibility. The results suggest, in particular, that a tendency for reflection increases with age, is stable over periods as long as twenty months, exhibits considerable generality across a varied assortment of task situations, and is linked to some fundamental aspects of the child's personality organization.

The results of the studies pertaining to problem-solving indicate that success in mathematical problem-solving is influenced by factors such as I. Q., reading ability, grades in previous mathematics courses, ability to determine spatial relations, ability to resist distractions, ability to identify critical elements, and ability to disregard irrelevant elements. Since many of these characteristics could also affect performance on the Matching Familiar Figures Test, a possible link between conceptual tempo and mathematical

problem-solving may be hypothesized on the basis of these studies. Furthermore, the studies by Pace (1961) and Klieblian (1955) support the choice of understanding arithmetic operations and estimation as valid aspects of problem-solving to be investigated.

Several researchers (Lee, Kagan, and Rabson, 1963) subscribe to the belief that the reflective-impulsive response dimension has value especially in disciplines with deductive structures such as mathematics and physical science. In all the studies on conceptual tempo that were reviewed, except for that of Callahan and Passi (1971), the performance tasks dealt with nonmathematical areas such as word and picture recognition, recall of serially learned material, and generalized inductive reasoning. It is hoped that the present study will contribute additional information about the conceptual tempo of children in relation to their mathematical problem-solving ability.

CHAPTER III

THE EXPERIMENTAL DESIGN

Statement of Hypotheses

In view of the results of the preceding chapter, it is also reasonable to hypothesize that the reflection-impulsivity dimension will generalize to arithmetic problem-solving tasks. It could be reasoned that the impulsive children will make more errors in estimation and in selecting the appropriate arithmetic operation because they do not pause to evaluate critically the quality of their responses. Impulsive children tend to respond quickly in situations involving response uncertainty; they report the first reasonable idea that occurs to them and are often incorrect. On the other hand, it could be conjectured that the reflective children, considering the differential validity of alternative answers, might prove to be more accurate estimators and might have less difficulty in selecting the proper arithmetic operation, especially in verbal problems such as those mentioned on page six.

Hence, the following null hypotheses will be investigated:

H1. There is no significant difference between the mean error scores of impulsive and reflective

subjects on those test questions involving elementary number operations.

H2. There is no significant difference between the mean error scores of impulsive and reflective subjects on those test questions pertaining to estimation.

H3. There is no significant difference between the mean error scores of impulsive and reflective subjects on the entire problem-solving test.

H4. There is no significant difference between the mean error scores of impulsive high achievers and reflective high achievers on those test questions involving elementary number operations.

H5. There is no significant difference between the mean error scores of impulsive high achievers and reflective high achievers on those test questions pertaining to estimation.

H6. There is no significant difference between the mean error scores of impulsive high achievers and reflective high achievers on the entire problem-solving test.

H7. There is no significant difference between the mean error scores of impulsive low achievers and reflective low achievers on those questions involving elementary number operations.

H8. There is no significant difference between the mean error scores of impulsive low achievers and reflective low achievers on those test questions pertaining to estimation.

H9. There is no significant difference between the mean error scores of impulsive low achievers and reflective low achievers on the entire problem-solving test.

H10. There is no significant difference between the mean error scores of high achievers and low achievers on those test questions involving elementary number operations.

H11. There is no significant difference between the mean error scores of high achievers and low achievers on those test questions pertaining to estimation.

H12. There is no significant difference between the mean error scores of high achievers and low achievers on the entire problem-solving test.

H13. There is no significant difference between the mean error scores of impulsive high achievers and impulsive low achievers on those test questions involving elementary number operations.

H14. There is no significant difference between the mean error scores of impulsive high achievers and impulsive low achievers on those test questions pertaining to estimation.

H15. There is no significant difference between the mean error scores of impulsive high achievers and impulsive low achievers on the entire problem-solving test.

H16. There is no significant difference between the mean error scores of reflective high achievers and reflective low achievers on those test questions involving elementary number operations.

H17. There is no significant difference between the mean error scores of reflective high achievers and reflective low achievers on those test questions pertaining to estimation.

H18. There is no significant difference between the mean error scores of reflective high achievers and reflective low achievers on the entire problem-solving test.

H19. There are no significant differences among the mean error scores, on those test questions involving elementary number operations, of the four subject groups formed by taking all possible pairings of response style with achievement status.

H20. There are no significant differences among the mean error scores, on those test questions pertaining to estimation, of the four subject groups formed by taking all possible pairings of response style with achievement status.

H21. There are no significant differences among the mean error scores, on the entire problem-solving test, of the four subject groups formed by taking all possible pairings of response style with achievement status.

H22. There is a constant difference between the mean error scores of impulsive and reflective subjects in the two achievement categories on those test questions involving elementary number operations.

H23. There is a constant difference between the mean error scores of impulsive and reflective subjects in the two achievement categories on those test questions pertaining to estimation.

H24. There is a constant difference between the mean error scores of impulsive and reflective subjects in the two achievement categories on the entire problem-solving test.

H25. There is a constant difference between the mean error scores for achievement categories within the two response style categories on those test questions involving elementary number operations.

H26. There is a constant difference between the mean error scores for achievement categories within the two response style categories on those test questions pertaining to estimation.

H27. There is a constant difference between the mean error scores for achievement categories within the two response style categories on the entire problem-solving test.

Description of Procedures and Design

The design of the present study can be classified under the broad heading of quasi-experimental research and takes the specific form of a 2x2 factorial experiment. The two experimental factors are conceptual tempo (reflective, impulsive) and achievement status (high, low) while the criterion measure is given by the error scores of the subjects on a series of problem-solving tasks involving elementary number operations and estimation.

For the purpose of the present study, three elementary schools were chosen from among the public elementary schools of Alachua County, Florida, and the combined fourth-grade enrollment of all three schools formed the experimental

population of the study. The schools were selected on the basis of the willingness of the principal and teachers to participate in the study. All three of the schools used were racially balanced. Two achieved this balance by means of busing while the location of the third produced this effect naturally. In addition, two different instructional arrangements were in use. Two schools were built on a 'quad', or open classroom, design, with the four rooms in each quad separated from each other by movable partitions. In these two schools, considerable pooling and interchanging of students and teachers took place for various subjects during the day. The third school was patterned along the more traditional lines of the self-contained classroom. Little interaction took place between classes for instructional purposes.

In October, 1971, as part of a county-wide testing program, the Stanford Achievement Test had been administered to all fourth-grade students in the three schools. The results on the Arithmetic Test, Intermediate I, Form X, were now used to determine the achievement status of the subjects. Those students whose stanine score on any two of the three parts of the Stanford Arithmetic Achievement Test was five or above, and whose stanine score was three or above on the remaining part, were designated as high achievers. Those students whose stanine scores on any two of the three parts of the Stanford Arithmetic Achievement Test was three or below, and whose stanine score was four or below on the

remaining part were classified as low achievers. This classification resulted in an available pool of 110 high achievers and 124 low achievers. From these two pools, random samples of 63 high achievers and 63 low achievers were drawn.

In April, 1972, a performance test on two components of general mathematical problem-solving ability, designed by the author and described in the next section, was administered to the entire fourth-grade population of all three schools. The variable recorded for each student on this test was the number of incorrect responses on each of the two component parts, and this served as the criterion measure for the experiment.

In May, 1972, the Matching Familiar Figures (MFF) Test, a conceptual tempo assessment scale, also to be described in the next section, was administered to the two groups of 63 high achievers and 62 low achievers previously selected. (One subject in the low achiever group was lost when his family moved from the area.) On the basis of their MFF scores, subjects may be categorized as impulsive, reflective, or neutral. The neutral subjects were deleted from the present study, so that four distinct, mutually exclusive subject groupings emerged from this selection process: impulsive high achievers, impulsive low achievers, reflective high achievers, and reflective low achievers. The cell distribution is shown in Table I. The 93 subjects in the four categories constitute the actual sample for the study.

In order to minimize any awareness on the part of the 93 subjects of their participation in an experiment, the problem-solving performance test was administered to all fourth-graders in the three schools. In actuality, however, only the scores of the 93 subjects previously selected for the four categories were used for the study.

TABLE I: Distribution of Experimental Subjects

	High	Low	Total
Impulsive	17	36	53
Reflective	27	13	40
Total	44	49	93

Instrumentation

This section is devoted to an examination of the three tests utilized in the present study.

Stanford Arithmetic Achievement Test

As indicated in the preceding section, the achievement classification of the subjects was effected by the use of the arithmetic subtest of the Stanford Achievement Test. Three levels of the test exist, but for the present study, only Intermediate Level I, Form X, 1964 Edition, (grades 4.0--5.5) was utilized. The test is administered in two sessions each approximately an hour and a half in length, and yields three scores: computations, concepts, and applications.

The arithmetic computations subtest measures proficiency in the computational skills appropriate for grade four and the beginning of grade five. The 39 computation items are drawn from the fundamental operations of addition, subtraction, multiplication, and division. The test is in multiple-choice form, and a generous time limit reduces the emphasis on computational speed. The exercises are representative of the traditional curriculum and textbook patterns of content.

The arithmetic concepts subtest consists of 32 items designed to measure the student's understanding of topics such as place value, Roman numerals, operational terms, meaning of fractions, and the interrelationship of the two fundamental operations of addition and multiplication, and their inverse operations of subtraction and division.

The arithmetic applications section has 33 multiple-choice items designed to measure reasoning ability with problems from life experiences. The general reading vocabulary is far below the problem-solving level being measured, and computational difficulty has been controlled. The student is required to apply his mathematical knowledge and thinking ability to practical situations involving area, volume, ratio, graphs, scales, percent, averages, and business transactions.

The Stanford Arithmetic Achievement test was standardized on 850,000 students in 264 school systems in all 50 states. The test was administered from February 25 to

March 15, 1963. The rigorous analysis of test items by the authors helps to insure content validity. The split-half reliability coefficients, corrected by the Spearman-Brown prophecy formula, for the computations, concepts, and applications section are 0.88, 0.86, and 0.86 respectively.

Matching Familiar Figures Test

The Matching Familiar Figures (MFF) instrument is a visual matching test developed by Dr. Jerome Kagan. It consists of two practice items and twelve test items and is administered individually to each subject. For each item, the subject is simultaneously presented with a picture (the standard) and six similar stimuli, only one of which is identical to the standard. The subject's task is to select the variant that matches the standard unit. If the subject's first response is incorrect, he is told that he is incorrect and he tries again. The procedure is repeated until a correct response is made or the subject makes six incorrect responses. The subject is not permitted to keep a written record of his choices and should an incorrect choice be repeated, it is again counted as an incorrect response. Memory requirements are eliminated as the standard and the variations are always simultaneously available to the subject. Since the response variations for each item differ only slightly from the standard, the test presents a situation in which response uncertainty is present. The variables scored are the total number of errors and the average response time to first selection across the twelve test items.

In the classification of subjects as impulsive or reflective by means of the MFF test, the usual procedure is to employ a split median analysis on the two variables, average response latency and total errors. Subjects within a group who are above the median on latency and below the median on errors are categorized as reflective. Those subjects within a group who are below the median on latency and above the median on errors are classified as impulsive. The remainder of the subjects fall into a neutral category. Previous studies by Kagan, Pearson, and Welch (1966) and Callahan and Passi (1971) indicate that a distribution of approximately one-third reflective, one-third impulsive, and one-third neutral can be expected in a tested group. For the purpose of the present study, the 63 high achievers and the 62 low achievers were treated as separate groups, and in each case, the subjects within the neutral category were discarded. As Table I indicates, the distribution of high achievers approximates the expected distribution rather closely, while the low achiever distribution differs considerably, especially in the impulsive category. The MFF instrument was administered to all 125 subjects by the author of the study.

Problem-solving Test

The performance test in the present study consists of a series of questions designed to assess certain aspects of the problem-solving behavior of fourth-grade students. Section A deals with a student's ability to analyze a verbal

arithmetic problem and decide which one of the four basic arithmetic operations is involved in the solution. The twenty items comprising this part of the test are all one-step verbal arithmetic problems, with five problems for each of the operations of addition, subtraction, multiplication, and division. Furthermore, several of the items are written in a nonnumerical format so that the student's performance can be determined in the absence of any cues which might be supplied by numerical values in the problem. All twenty problems have been assigned a position in the test by means of a table of random numbers so that pooling of problem types has been avoided. Although the reading level has been carefully structured to be appropriate to fourth-grade students, any reading bias is further diminished by having the examiner read each question aloud while the students follow along on their individual copies. Sufficient time is allotted after each question for the students to mark their multiple choice answer sheets. In addition, the students are instructed to request more time if needed. Students are not asked to solve any problem, but to simply decide which operation is involved.

Section B consists of twelve items relating to the student's ability to estimate in a variety of situations. The first eight questions assess the student's ability to estimate length, area, volume, and number when a known standard is presented in conjunction with the object or quantity to be estimated. The units of measurement specified

for each of the standards are abstract terms, so that any advantages due to familiarity with the usual standard units of measurement are reduced. The remaining four items are concerned with the student's ability to estimate the solution of a problem involving arithmetic computation. All answers to Section B of the test are multiple choice, and the students are instructed in each case, to select the answer which provides the closest estimate of the quantity involved.

The problem-solving test was administered to the students in two classes at the P. K. Yonge laboratory school in March, 1972. Each class was composed of an equal number of third-, fourth-, and fifth-grade students, and the combined membership total was 55 students. The results of this trial run were subjected to a difficulty test using the following criterion: If X represents the number of correct responses to a particular question, then the question is judged to be acceptable only if $.10N < X < .90N$, where N represents the total number of students taking the test. According to this formula, three questions failed to discriminate adequately, and were rewritten. In addition, a split-half reliability coefficient was calculated for the test on the basis of the Spearman-Brown prophecy formula. The value of r_{tt} was 0.85.

The revised problem-solving test was then administered by the author to all fourth-graders in the three schools involved in the study. Testing sessions lasted

approximately 45 minutes. The students were tested in classroom units, or, in the case of a quad design, in pairs of adjoining classroom units with the separating partition removed. Each student received an answer sheet and a copy of the verbal arithmetic problems in Section A of the test. General instructions concerning the mechanics of the test were given orally by the examiner, and students were assured that the test results would in no way influence their school grades. The examiner read each question in Section A twice, with the students following along on their question sheets. Sufficient time was allotted for the students to mark their multiple-choice answer sheets. A short break followed at the end of Section A while the question sheets were collected and oral instructions for Section B were given. Copies of Section A and Section B of the performance test may be found in Appendix A.

Statistical Treatment

The data gathered in the present study were analyzed using the system of applied multiple linear regression described by Bottenberg and Ward (1963). This particular use of multiple regression as a general approach to the formulation and analysis of research problems provides a technique which is powerful, but less restrictive than multivariate correlation procedures. Among its advantages is the elimination of the need to match experimental subjects or equate cell frequencies. In general, the investigator

is permitted to tailor the analytical procedures to the particular research question, rather than having to fit the study into a predetermined "standard" design.

Following the procedures of Bottenberg and Ward, a full mathematical model was formulated for each hypothesis in the study. The hypothesis was then reformulated as a series of mathematical restrictions, which were then imposed on the full model to obtain a restricted model. A multiple regression analysis computer program called PERSUB was employed to compute the error sum of squares for each of the two models, and the F-statistic resulting from their comparison. This F-value was then compared with tabled values to determine whether to accept or reject the hypothesis at a predetermined level of confidence.

CHAPTER IV

ANALYSIS OF DATA

The first three hypotheses involve a comparison of impulsive subjects and reflective subjects without regard to achievement status. The mean error scores for these hypotheses are found in Table II, while F-ratios and significance levels appear in Table III.

TABLE II: Mean Error Scores of Impulsive and Reflective Subjects on Section A, Section B, and Entire Test

	Impulsive	Reflective
Section A	10.68	5.98
Section B	8.89	8.95
Entire Test	19.57	14.92

H1. There is no significant difference between the mean error scores of impulsive and reflective subjects on those test questions involving elementary number operations.

H2. There is no significant difference between the mean error scores of impulsive and reflective subjects on those test questions pertaining to estimation.

H3. There is no significant difference between the mean error scores of impulsive and reflective subjects on the entire problem-solving test.

TABLE III: F-ratios and Significance Levels --- Impulsive Subjects Compared With Reflective Subjects

	F	Significance Level
Section A	22.9243	$p < 0.001$
Section B	0.0375	$p = 0.847$
Entire Test	17.0422	$p < 0.001$

Hypotheses H1-H3 state that there is no difference in the mean error scores of impulsive and reflective subjects. If these hypotheses are in fact true, then differences as large or larger than those obtained could occur by chance on Section A and the entire test less than 0.1 percent of the time. Such differences on Section B could occur 84.7 percent of the time. In the comparison of the impulsive and reflective groups on Section A and the entire test, the F-ratio is larger than that needed for significance at the 0.01 level of confidence. The F-ratio for the comparison on Section B is less than that needed for significance at the 0.05 level. Hence, null hypotheses H1 and H3 can be rejected, while H2 cannot. This means that reflective subjects did significantly better than impulsive subjects on Section A and the entire test. The results on Section B, although not significant at the 0.05 level, indicate that the impulsive subjects apparently did slightly better than the reflective subjects.

Hypotheses H4-H6 involve a comparison of impulsive

subjects and reflective subjects within the high achiever category. The mean error scores for these hypotheses are given in Table IV, while F-ratios and significance levels appear in Table V.

TABLE IV: Mean Error Scores of Impulsive High Achievers and Reflective High Achievers on Section A, Section B, and Entire Test

	Impulsive High Achievers	Reflective High Achievers
Section A	5.76	4.07
Section B	8.06	8.67
Entire Test	13.82	12.74

H4. There is no significant difference between the mean error scores of impulsive high achievers and reflective high achievers on those test questions involving elementary number operations.

H5. There is no significant difference between the mean error scores of impulsive high achievers and reflective high achievers on those test questions pertaining to estimation.

H6. There is no significant difference between the mean error scores of impulsive high achievers and reflective high achievers on the entire problem-solving test.

TABLE V: F-ratios and Significance Levels --- Impulsive High Achievers Compared With Reflective High Achievers

	F	Significance Level
Section A	2.4172	$p = 0.124$
Section B	1.7377	$p = 0.191$
Entire Test	0.7774	$p = 0.380$

Hypotheses H4-H6 state that there is no difference in the mean error scores of impulsive high achievers and reflective high achievers. If these hypotheses are in fact true, then differences as large or larger than those observed could occur by chance on Section A, Section B, and the entire test, from 12.4 percent to 38.0 percent of the time. None of the F-ratios in Table V exceeds that required for significance at the 0.05 confidence level, and hence none of the null hypotheses H4-H6 can be rejected. This indicates that within the high achievers category, there is no significant difference between the mean error scores of impulsive and reflective subjects. A comparison of the means shows that, although the results did not reach significance at the 0.05 level, reflective high achievers apparently did better than impulsive high achievers on Section A and on the entire test, while impulsive high achievers seemed to do slightly better than reflective high achievers on Section B.

Hypotheses H7-H9 involve a comparison of impulsive

subjects and reflective subjects within the low achiever category. The mean error scores for these hypotheses are given in Table VI, while F-ratios and significance levels appear in Table VII.

TABLE VI: Mean Error Scores of Impulsive Low Achievers and Reflective Low Achievers on Section A, Section B, and Entire Test

	Impulsive Low Achievers	Reflective Low Achievers
Section A	13.00	9.92
Section B	9.28	9.54
Entire Test	22.28	19.46

H7. There is no significant difference between the mean error scores of impulsive low achievers and reflective low achievers on those questions involving elementary number operations.

H8. There is no significant difference between the mean error scores of impulsive low achievers and reflective low achievers on those test questions pertaining to estimation.

H9. There is no significant difference between the mean error scores of impulsive low achievers and reflective low achievers on the entire problem-solving test.

TABLE VII: F-ratios and Significance Levels --- Impulsive Low Achievers Compared With Reflective Low Achievers

	F	Significance Level
Section A	7.3305	$p = 0.008$
Section B	0.2925	$p = 0.590$
Entire Test	4.8152	$p = 0.031$

The null hypotheses H7-H9 assert that there is no difference in the mean error scores of impulsive and reflective subjects within the low achiever category. If these hypotheses are true, then differences as large or larger than those obtained could occur by chance 0.8 percent of the time on Section A, 59.0 percent of the time on Section B, and 3.1 percent of the time on the entire test. In the comparison of impulsive low achievers and reflective low achievers on Section A and the entire test, the F-ratios both exceed the value needed for significance at the 0.05 level of confidence and hence null hypotheses H7 and H9 can be rejected. The F-ratio for Section B, however, is less than that needed for rejection at the 0.05 level, and hence hypothesis H8 cannot be rejected. This indicates that reflective low achievers did significantly better than impulsive low achievers on Section A and the entire test. The results on Section B, although not significant at the 0.05 level, again indicate that the impulsive low achievers apparently did slightly better than the reflective low achievers.

Hypotheses H10-H12 relate to a comparison of high achievers and low achievers without regard to conceptual tempo level. The mean error scores for these hypotheses are found in Table VIII, while F-ratios and significance levels are listed in Table IX.

TABLE VIII: Mean Error Scores of High Achievers and Low Achievers on Section A, Section B, and Entire Test

	High Achievers	Low Achievers
Section A	4.73	12.18
Section B	8.43	9.35
Entire Test	13.16	21.53

H10. There is no significant difference between the mean error scores of high achievers and low achievers on those test questions involving elementary number operations.

H11. There is no significant difference between the mean error scores of high achievers and low achievers on those test questions pertaining to estimation.

H12. There is no significant difference between the mean error scores of high achievers and low achievers on the entire problem-solving test.

TABLE IX: F-ratios and Significance Levels --- High Achievers Compared With Low Achievers

	F	Significance Level
Section A	96.2924	$p < 0.001$
Section B	8.7505	$p = 0.004$
Entire Test	99.3543	$p < 0.001$

Hypotheses H10-H12 state that there is no difference in the mean error scores of high achievers and low achievers. If this is true, then differences of the same magnitude as those obtained could occur by chance on Section A and the

entire test less than 0.1 percent of the time. Differences on Section B could occur 0.4 percent of the time by chance. The F-ratios in Table IX all exceed the value required for significance at the 0.01 confidence level, and hence the null hypotheses H10-H12 must be rejected. Hence, as might be expected, high achievers did significantly better on Sections A and B, and the entire test.

Hypotheses H13-H15 involve a comparison between high achievers and low achievers within the impulsive category. The mean error scores for these hypotheses are found in Table X, while F-ratios and significance levels are listed in Table XI.

TABLE X: Mean Error Scores of Impulsive High Achievers and Impulsive Low Achievers on Section A, Section B, and Entire Test

	Impulsive High Achievers	Impulsive Low Achievers
Section A	5.76	13.00
Section B	8.06	9.28
Entire Test	13.82	22.28

H13. There is no significant difference between the mean error scores of impulsive high achievers and impulsive low achievers on those test questions involving elementary number operations.

H14. There is no significant difference between the mean error scores of impulsive high achievers and impulsive low achievers on those test questions pertaining to estimation.

H15. There is no significant difference between the mean error scores of impulsive high achievers and impulsive low achievers on the entire problem-solving test.

TABLE XI: F-ratios and Significance Levels --- Impulsive High Achievers Compared With Impulsive Low Achievers

	F	Significance Level
Section A	49.0051	$p < 0.001$
Section B	7.7357	$p = 0.007$
Entire Test	52.4631	$p < 0.001$

The three hypotheses H13-H15 assert that there is no difference in the mean error scores of impulsive high achievers and impulsive low achievers. Should these hypotheses be true, then differences as large or larger than those obtained can be expected to occur by chance alone less than 0.1 percent of the time on Section A and the entire test, and 0.7 percent of the time on Section B. An examination of Table XI indicates that all three F-ratios exceed the tabulated value needed for significance at the 0.05 level of confidence. Hence all three of the null hypotheses H13-H15 must be rejected. This means that within the impulsive category, the performance of high achievers was significantly better than that of low achievers on Sections A and B, and the entire test.

Hypotheses H16-H18 deal with the comparison of high

achievers and low achievers within the reflective category. The mean error scores for the hypotheses in question are found in Table XII, while the F-ratios and significance levels are presented in Table XIII.

TABLE XII: Mean Error Scores of Reflective High Achievers and Reflective Low Achievers on Section A, Section B, and Entire Test

	Reflective High Achievers	Reflective Low Achievers
Section A	4.07	9.92
Section B	8.67	9.54
Entire Test	12.74	19.46

H16. There is no significant difference between the mean error scores of reflective high achievers and reflective low achievers on those test questions involving elementary number operations.

H17. There is no significant difference between the mean error scores of reflective high achievers and reflective low achievers on those test questions pertaining to estimation.

H18. There is no significant difference between the mean error scores of reflective high achievers and reflective low achievers on the entire problem-solving test.

TABLE XIII: F-ratios and Significance Levels --- Reflective High Achievers Compared With Reflective Low Achievers

	F	Significance Level
Section A	24.3368	$p < 0.001$
Section B	3.0069	$p = 0.086$
Entire Test	25.1951	$p < 0.001$

Hypotheses H16-H18 claim that no difference exists in the mean error scores of high achievers and low achievers within the reflective category. If these are true, then differences as large or larger than those obtained can be expected to occur by chance alone less than 0.1 percent of the time on Section A and the entire test, and 8.6 percent of the time on Section B. An inspection of Table XIII shows that the F-ratios for Section A and the entire test exceed the tabulated value needed for significance at the 0.01 confidence level, while the F-ratio in Section B does not exceed that needed for significance at the 0.05 level. Hence hypotheses H16 and H18 can be rejected, while H17 cannot. This means that within the reflective category, high achievers did significantly better than low achievers on Section A and the entire test. Although the results were not significant at the 0.05 level, reflective high achievers also appeared to do better than reflective low achievers on Section B.

Hypotheses H19-H21 relate to comparisons of all

subject groups formed by taking all possible pairings of response style with achievement status. Table XIV contains F-ratios and significance levels. The four cell means have already been stated in Table X and Table XII.

H19. There are no significant differences among the mean error scores, on those test questions involving elementary number operations, of the four subject groups formed by taking all possible pairings of response style with achievement status.

H20. There are no significant differences among the mean error scores, on those test questions pertaining to estimation, of the four subject groups formed by taking all possible pairings of response style with achievement status.

H21. There are no significant differences among the mean error scores, on the entire problem-solving test, of the four subject groups formed by taking all possible pairings of response style with achievement status.

TABLE XIV: F-ratios and Significance Levels --- Simultaneous Comparison of All Four Subject Cells

	F	Significance Level
Section A	38.0795	$p < 0.001$
Section B	3.5946	$p = 0.017$
Entire Test	25.1951	$p < 0.001$

The null hypotheses H19-H21 assert that there are no statistically significant differences among the four cell means. If these hypotheses are really true, then differences comparable to or greater than those observed could occur by chance less than 0.1 percent of the time on

Section A and the entire test. On Section B, such differences could occur 1.7 percent of the time. All three F-ratios in Table XIV are large enough to reach significance at the 0.05 confidence level, and hence all three null hypotheses H19-H21 must be rejected. This indicates that there are significant differences in the error mean scores of subject groups having different patterns of attributes with respect to conceptual tempo and achievement status.

Hypotheses H22-H24 deal with the question of constant differences between impulsive and reflective subjects within the two achievement categories. Hypotheses H25-H27 deal with the question of constant differences between the two achievement categories within the impulsive and reflective groups. Both groups of hypotheses presuppose the same full model, and when the two sets of mathematical restrictions are imposed, the same restricted model is also produced. Hence, in this case, a single set of F-ratios and significance levels serves to test all six hypotheses. These values may be found in Table XV.

H22. There is a constant difference between the mean error scores of impulsive and reflective subjects in the two achievement categories on those test questions involving elementary number operations.

H23. There is a constant difference between the mean error scores of impulsive and reflective subjects in the two achievement categories on those test questions pertaining to estimation.

H24. There is a constant difference between the mean error scores of impulsive and reflective subjects in the two achievement categories on the entire problem-solving test.

H25. There is a constant difference between the mean error scores for achievement categories within the two response style categories on those test questions involving elementary number operations.

H26. There is a constant difference between the mean error scores for achievement categories within the two response style categories on those test questions pertaining to estimation.

H27. There is a constant difference between the mean error scores for achievement categories within the two response style categories on the entire problem-solving test.

TABLE XV: F-ratios and Significance Levels --- Constant Differences

	F	Significance Level
Section A	0.7768	p = 0.381
Section B	0.2708	p = 0.604
Entire Test	0.9524	p = 0.332

Hypotheses H22-H24 claim a constant difference between the mean error scores of impulsive and reflective subjects within the two achievement categories. If these hypotheses are true, than differences equal to or greater than those observed could occur by chance 38.1 percent of the time in Section A, 60.4 percent of the time in Section B, and 33.2 percent of the time on the entire test. All three F-ratios fail to attain the value needed for the 0.05 level of significance and hence all three null hypotheses H22-H24 cannot be rejected.

Hypotheses H25-H27 claim a constant difference between the mean error scores of high achievers and low

achievers within the two response style categories. If this is true, then differences equal to or greater than those observed could occur by chance 38.1 percent of the time in Section A, 60.4 percent of the time in Section B, and 33.2 percent of the time on the entire test. All three F-ratios fail to attain significance at the 0.05 level, and hence all three null hypotheses H25-H27 cannot be rejected.

During a planning session for the present study, a question was raised concerning the distribution of reflective subjects and impulsive subjects according to their performance in selecting the appropriate operation in verbal arithmetic problems, and in estimating. Accordingly, a count of subjects whose error scores on Section A was in either the upper one-third or the lower one-third of the distribution, and whose error score on Section B was in either the upper one-third or the lower one-third, was made without regard for conceptual tempo. The cell distribution is shown in Table XVI.

TABLE XVI: Cell Distribution of Subjects Whose Error Scores Rank in the Upper or Lower One-third on Sections A and B

		Section A -- Operations	
		Upper	Lower
Section B -- Estimation	Upper	12	11
	Lower	5	15

Similar distributions were prepared separately for the

impulsive and reflective groups. These cell distributions appear in Table XVII.

TABLE XVII: Cell Distribution of Impulsive and Reflective Subjects Whose Error Scores Rank in the Upper or Lower One-third on Section A and B

		Section A -- Operations			
		Impulsive		Reflective	
		Upper	Lower	Upper	Lower
Section B -- Estimation	Upper	7	5	4	5
	Lower	3	10	2	7

In order to obtain a measure of the relationship between the ability to determine the correct operation in a verbal arithmetic problem and the ability to estimate, the tetrachoric coefficient of correlation, r, was computed for Table XVI and for the impulsive and reflective portions of Table XVII. The value of $r = 0.43$ was obtained for Table XVI, while the impulsive and reflective parts of Table XVII yielded r values of 0.55 and 0.39 respectively.

Finally, in order to determine to what extent the cell distribution of impulsive subjects differed from that for reflective subjects, the data in Table XVII were subjected to a chi-square analysis. The computed value of the test statistic was 4.04, which did not exceed the tabulated value at the 0.05 level.

CHAPTER V

SUMMARY, CONCLUSIONS, LIMITATIONS, AND IMPLICATIONS

Summary

The present study was intended to investigate the relationship between conceptual tempo (a cognitive response style) and certain aspects of a fourth-grade student's problem-solving ability. The existence and significance of this dimension of reflection-impulsivity in problem situations had been experimentally verified by a series of research studies. Some information on the generality of conceptual tempo across tasks had been collected, but in almost all cases, the performance tasks had been of a non-mathematical nature. It was decided to examine conceptual tempo in a mathematical setting.

Since conceptual tempo was related to situations involving high response uncertainty and several alternative solution hypotheses, mathematical problem-solving was selected for the study. A review of the literature was made to determine the most appropriate aspects of problem-solving ability to be investigated. The two finally chosen were the ability to analyze a verbal arithmetic problem and select

the appropriate operation to be performed, and the ability to estimate. A performance test involving these two characteristics was constructed by the author, pilot-tested at the P. K. Yonge laboratory school, and then administered to the entire fourth-grade population of three public elementary schools in Alachua County, Florida. The error scores on Sections A and B of this test served as the criterion measures for the study. In the three schools tested, all fourth-grade students had been administered the arithmetic subtest of the Stanford Achievement Test, Intermediate I, Form X. On the basis of these scores, students were classified as high achievers, low achievers, or neither. From the first two groups, 63 high achievers and 63 low achievers were selected at random. These students were then given the Matching Familiar Figures Test, a conceptual tempo assessment scale, and were classified as impulsive or reflective on the basis of their error scores. The 93 students in the four categories of impulsive high achiever, impulsive low achiever, reflective high achiever, and reflective low achiever formed the sample for the study.

The mean error scores for the four groups were compared in a 2x2 factorial design using a system of applied multiple linear regression analysis outlined by Bottenberg and Ward (1963). A multiple regression analysis computer program called PERSUB was used to calculate F-ratios for the 27 null hypotheses. The error score distributions of impulsive and reflective subjects on each of the two Sections A

and B were divided into thirds. The cell distributions for each of the two response styles over the following four categories were then determined: (1) upper-third Section A and upper-third Section B, (2) upper-third Section A and lower-third Section B, (3) lower-third Section A and upper-third Section B, and (4) lower-third Section A and lower-third Section B. The two cell distributions were then compared by means of a chi-square analysis.

Conclusions

The following conclusions may be drawn from the study:

1. In a comparison of impulsive and reflective subjects without regard for achievement, the reflective subjects did significantly better than the impulsive subjects on the entire test and on Section A, which involved identification of the operation in a verbal arithmetic problem. On Section B, which involved estimation, there was no significant difference between impulsive and reflective subjects. However, impulsive subjects appeared to do slightly better than reflective subjects.
2. In a comparison of impulsive and reflective subjects within the high achiever category, there was no significant difference between impulsive high achievers and reflective high achievers on Section A, Section B, or the entire test. Although the results were not significant, there was an indication that reflective high

achievers did better on Section A and the entire test, and impulsive high achievers appeared to do slightly better on Section B.

3. In a comparison of impulsive and reflective subjects within the low achiever category, reflective low achievers did significantly better than impulsive low achievers on Section A and on the entire test. There was no significant difference between the two groups on Section B, although it would appear that impulsive low achievers did do slightly better.
4. In a comparison of high achievers and low achievers without regard to conceptual tempo, high achievers did significantly better than low achievers on Section A, Section B, and the entire test.
5. In a comparison of high achievers and low achievers within the impulsive category, the performance of impulsive high achievers was significantly better than that of impulsive low achievers on Section A, Section B, and the entire test.
6. In a comparison of high achievers and low achievers within the reflective category, reflective high achievers did significantly better than reflective low achievers on Section A and the entire test. There was no significant difference between the two groups on Section B, although it appeared that reflective high achievers did slightly better.
7. There were significant differences between subject

groups having different patterns of attributes with respect to conceptual tempo and achievement.

8. The interaction of conceptual tempo and achievement status did not make a statistically significant difference in error scores on Section A, Section B, and the entire test.

9. In a grouping of subjects whose error scores on both of the Sections A and B are either in the upper or lower one-third of the distribution, a slight (tetrachoric $r = 0.43$) correlation exists between ability to identify an operation and ability to estimate. Slight ($r = 0.39$) and moderate ($r = 0.55$) correlations exist in similar distributions done respectively for reflective subjects only and impulsive subjects only. There was no significant difference in the two distributions for impulsive and reflective subjects among these four categories.

In general, the results of the present study support the results of Callahan and Passi (1971), Kagan, Rosman, Day, Albert and Phillips (1964), Kagan (1965b), and Kagan, Pearson and Welch (1966) in that reflective subjects did better (i.e. had smaller error scores) than impulsive subjects in the particular performance task in question. In the various comparisons of impulsive subjects and reflective subjects on Section B of the test, the results did not usually achieve statistical significance. Any of the apparent differences observed, however, always involved a slightly smaller error mean score for the impulsive subjects. Hence there is a

need for another related study to investigate the conjecture that in situations involving estimation, a reflective response style may actually reduce problem-solving effectiveness. In addition, a deviation was noted from the one-third reflective, one-third impulsive, and one-third neutral distribution of subjects on the Matching Familiar Figures Test observed by Kagan, Pearson and Welch (1966) and Callahan and Passi (1971).

Limitations

Some of the limitations of the study are as follows:

1. The three schools involved in the study were not alike in terms of the general socioeconomic background of the students.
2. The estimation error scores of students sitting at the rear of a classroom may have been influenced by their distance from the examiner.
3. Although most of the reading bias may have been removed by having the examiner read the questions aloud, a generally poor level of verbal skills may have influenced the error scores of some students.

Implications

The present study contributes to the growing body of information about the reflection-impulsivity dimension of children, especially as it pertains to mathematical

abilities. Such information should be useful to the mathematics teacher in helping his students establish permanent attitudes toward problem-solving and strategies of solution with novel problem tasks. In complex problem situations with many alternate solution possibilities, reflection over these alternatives determines the ease with which a student arrives at the solution. The impulsive student is apt to implement the first idea that occurs to him, and is likely to be wrong more often. Such a failure increases his anxiety and impairs his selection of a second solution strategy. This process repeats itself, and after years of experiencing this sequence, the impulsive child may avoid involvement in problem situations and become apathetic or hostile toward intellectual situations. The mathematics teacher who is familiar with the conceptual tempo dimension can identify such a student and encourage him to make more use of reflection in his problem-solving attempts.

Finally, it is hoped that this study will encourage other studies of conceptual tempo in mathematical settings. Studies dealing with the interaction of conceptual tempo and classroom presentation, the feasibility of matching students and teachers on conceptual tempo, and the long-term effects of training in reflection could provide the mathematics teacher with much useful knowledge.

APPENDIX

PROBLEM-SOLVING TEST

SECTION A

1. Mary, Susan and Jane helped their mother make sandwiches for a church picnic. Mary and Susan together made 137 sandwiches, while Jane made 46 sandwiches. How many sandwiches did the girls make?

2. Bill and his father drove 58 miles from their home to Centerville. From there they drove 153 miles to Lake Jackson. How much farther was it from Centerville to Lake Jackson then from home to Centerville?

3. Mr. Thomas paid $2.94 for some pads of paper for his office. The pads cost $.07 each. How many pads did Mr. Thomas buy?

4. Joe read three library books. One was 240 pages long, one was 318 pages long, and one was 173 pages long. How many pages did Joe read?

5. Mary has some cookies and wants to share them with a group of her friends. She would like for each friend to have the same number of cookies. If Mary knows how many cookies she has and how many friends are to share them, how can she find out how many cookies to give each friend?

6. David is in charge of the refreshments for the class party. If each student is to get one piece of cake and there are 24 girls and 19 boys, how many pieces of cake will David need?

7. Betty earned some money by baby-sitting for Mrs. Jones on 6 different mornings. In all Betty was paid $5.10. How much did she receive for each morning's work?

8. Mr. Johnson bought 9 rolls of film for his new camera. The camera can take 12 pictures with each roll of film. If he uses all 9 rolls of film, how many pictures can Mr. Johnson take?

9. A baker made 54 cupcakes. He put the cakes into boxes, with 6 cakes in each box. How many boxes did he fill?

10. At its grand opening, the Redtop Market gave away a free balloon to every boy and girl who visited the market. In the morning, the store gave away 183 balloons. That afternoon 327 balloons were given away. How many more balloons were given away in the afternoon than in the morning?

11. Mr. Green raises tomatoes on his farm. This year, he has 308 tomato plants which he has planted in 11 rows. How many tomato plants are there in each row?

12. Harry's mother gave him some money when he left for school. During the day, he lost some of it. When he got home, his mother counted the money he had left. How could Harry's mother find out how much money he lost?

13. Eric and his brother Tom collect marbles. They keep the marbles in small boxes, and every box has the same number of marbles. If the boys know how many marbles are in a box and how many boxes they have, how can they find out how many marbles they have in their collection?

14. Paul has a job at the grocery store after school. He makes $2.25 a day. How much money will Paul have made after he has worked 8 days?

15. Ann bought 6 packets of stamps, with 35 stamps in each packet. How many stamps did she receive in all?

16. Ted knows how many baseball cards he has and how many his brother Bill has. If Bill has more cards than Ted, how can Ted find out how many cards he will have if he gets all of Bill's cards?

17. Sally was one of the helpers at a PTA baked goods sale. Her mother baked 5 cakes for the sale. Sally saw that the cakes were priced at $1.20 each. How much money did the five cakes sell for?

18. Mary's Girl Scout troop is selling cookies. The first girl to sell 75 boxes will receive a prize. Mary has sold 53 boxes so far. How many more boxes does Mary need to sell in order to win the prize?

19. At the county fair, Mary paid $.35 for a bag of popcorn, $.55 for some cotton candy, and $.75 for a ride on the roller coaster. If she had $2.85 when she arrived, how much money did Mary spend at the fair?

20. Mrs. Johnson's fourth grade class sold candied apples at the school carnival. They started with 109 apples. After the carnival they had 35 apples left. How many apples did they sell?

SECTION B: DESCRIPTION OF QUESTIONS

1. Students are shown, as a standard, a small piece of dowel approximately four inches long. They are instructed to consider this stick as representing one gizmo. Using this as a standard, they are asked to estimate the examiner's height (5'11") in gizmos.

2. Standard same as above. In this instance, students are asked to estimate, in gizmos, the length of a piece of string approximately 10 feet long, stretched across the front of the room.

3. Students are shown, as a standard, a small piece of cardboard approximately one inch square. They are instructed to consider the square as representing one gronk. Using this, they are asked to estimate the number of gronks needed to equal the surface area of a card approximately eight inches square.

4. Standard same as above. The task here is to estimate the surface area, in gronks, of an irregular curved region of approximately 80 square inches, drawn on a piece of cardboard.

5. Students are shown, as a standard, a small block of wood approximately one cubic inch in volume. They are instructed to consider the cube as representing one gink. Using this, they are asked to estimate the volume, in ginks, of a larger cube approximately $4\frac{1}{2}$ inches on a side.

6. Standard same as above. Students are asked to estimate in ginks, the volume of an irregularly shaped block of wood.

7. Students are shown, as a standard, a card containing twenty-five dots of approximately $\frac{1}{2}$ inch diameter each. Using this as a guide, they are then asked to estimate the number of such dots which appear on a large piece of cardboard. This sheet contains 1200 such dots, arranged symmetrically in rows and columns.

8. Students are shown, as a standard, a small plastic bag containing fifty navy beans. Using this as a guide, they are asked to estimate the number of such beans contained in a jar. (1800)

9. $\frac{29 + 31}{30} =$

10. $19 + 11 + 21 =$

11. $39 \times 21 =$

12. $584 - 295 =$

(Problems 9-12 are presented to the students via flash cards.)

SAMPLE ANSWER SHEET

NAME: _____ AGE: ____ GRADE: _____

1.	+	-	x	÷
2.	+	-	x	÷
3.	+	-	x	÷
4.	+	-	x	÷
5.	+	-	x	÷
6.	+	-	x	÷
7.	+	-	x	÷
8.	+	-	x	÷
9.	+	-	x	÷
10.	+	-	x	÷
11.	+	-	x	÷
12.	+	-	x	÷
13.	+	-	x	÷
14.	+	-	x	÷
15.	+	-	x	÷
16.	+	-	x	÷
17.	+	-	x	÷
18.	+	-	x	÷
19.	+	-	x	÷
20.	+	-	x	÷

1.	12	15	18	21	24
2.	25	30	35	40	45
3.	15	30	45	65	100
4.	30	45	60	80	100
5.	35	50	65	105	210
6.	35	55	70	110	250
7.	900	1200	1500	1800	2100
8.	900	1200	1500	1800	2100
9.	10	8	6	4	2
10.	15	30	50	75	90
11.	650	800	950	1000	1200
12.	280	320	360	400	450

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BIOGRAPHICAL SKETCH

Bruce Peter Walek was born March 7, 1941, in Worcester, Massachusetts. At an early age, he moved to Miami, Florida, where he attended the Dade County public schools. He graduated from Miami Senior High School in June, 1959, and in September of that same year, enrolled at the University of Florida. While there, he persued an undergraduate program in mathematics and received the degree of Bachelor of Science, With High Honors, in June of 1963. For the next two years, he held the position of Graduate Teaching Assistant in the mathematics department of the University of Florida. In August, 1965, he received the degree of Master of Science in mathematics. From September, 1965, until June, 1970, he taught undergraduate mathematics at the University of Florida as a Graduate Teaching Assistant and as an Interim Instructor. In September of 1970, he was awarded a one-year Graduate School Fellowship. Since September of 1971, he has been employed at Santa Fe Community College while completing the requirements for the degree of Doctor of Philosophy with specialization in mathematics education.

Bruce Peter Walek is a member of Phi Kappa Phi, Phi Delta Kappa, Kappa Delta Pi, the Mathematical Association of America, and the National Council of Teachers of Mathematics.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Dr. Kenneth P. Kidd, Chairman
Professor of Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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